Halo clustering with f_{NL}, g_{NL} and τ_{NL}

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Michigan, May 2011

Smith & LoVerde, 1010.0055
LoVerde & Smith, 1102.1439
Smith, Ferraro & LoVerde, to appear

Generalized local non-Gaussianity

Primordial non-Gaussianity defined by:

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}(\Phi_G(\mathbf{x})^2 - \langle\Phi_G^2\rangle) + g_{NL}(\Phi_G(\mathbf{x})^3 - 3\langle\Phi_G^2\rangle\Phi_G(\mathbf{x}))$$

Possible mechanisms:

- curvaton scenario (spectator field during inflation subsequently dominates energy density)
- models with variable inflaton decay rate
- models with modulated reheating
- multifield ekpyrotic models (e.g. "New Ekpyrosis")

Generalization: f_{NL}-type model in which amplitude of 3-point function (f_{NL}) and amplitude of 4-point function (τ_{NL}) are independent parameters

$$\langle\zeta_{\mathbf{k_1}}\zeta_{\mathbf{k_2}}\zeta_{\mathbf{k_3}}\rangle = \frac{6}{5}f_{NL}\Big[P_\zeta(k_1)P_\zeta(k_2) + \text{cyc.}\Big](2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

$$\langle\zeta_{\mathbf{k_1}}\zeta_{\mathbf{k_2}}\zeta_{\mathbf{k_3}}\zeta_{\mathbf{k_4}}\rangle = \tau_{NL}\Big[P_\zeta(k_1)P_\zeta(k_2)P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|) + 11\text{ perm.}\Big](2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$$

[Normalization is defined so that "standard" f_{NL} cosmology corresponds to $\tau_{NL} = \left(\frac{6}{5}f_{NL}\right)^2$]

Generalized local non-Gaussianity

Simple model in which $\tau_{NL} \neq \left(\frac{6}{5} f_{NL}\right)^2$: (Tseliakhovich & Hirata 2010)

Initial potential is linear combination of two fields: $\Phi = (1-\alpha^2)^{1/2}\Phi_i + \alpha\Phi_c$

where Φ_i is Gaussian and Φ_c has f_{NL}-type non-Gaussianity $(\Phi_c = \Phi_G + f^c_{NL}(\Phi_G^2 - \langle\Phi_G^2\rangle))$

$$\left[\text{where } \alpha = \left(\frac{\tau_{NL}}{(6f_{NL}/5)^2}\right)^{-1/2}, \quad f^c_{NL} = \frac{f_{NL}}{\alpha^3}\right]$$

Scope of talk:

- Study halo clustering in non-Gaussian N-body simulations with parameter space $\{f_{NL}, g_{NL}, \tau_{NL}\}$
- Can also study mass function (companion talk by Marilena LoVerde)

Local non-Gaussianity: halo clustering

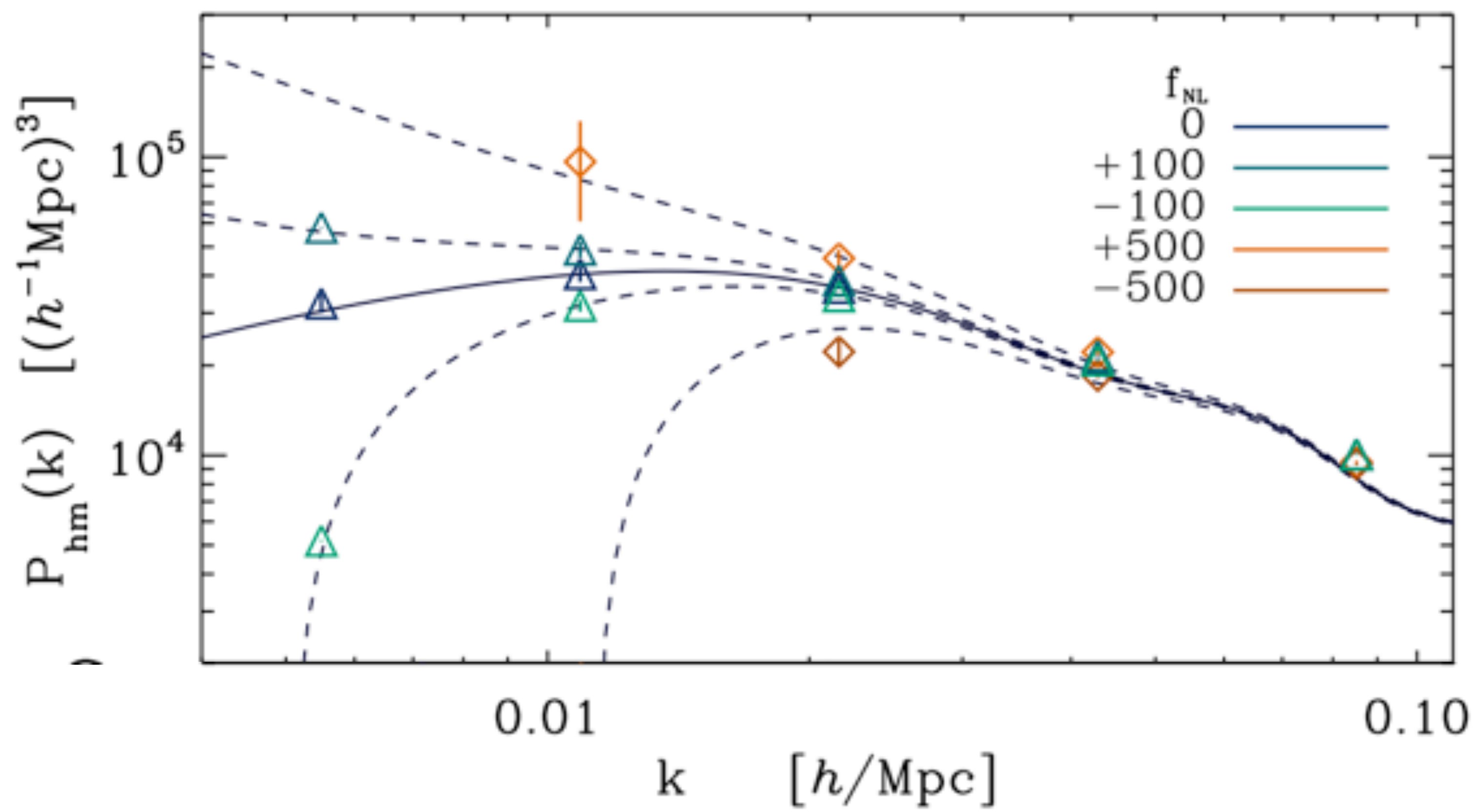
Dalal et al (2007): extra halo clustering on large scales in an f_{NL} cosmology

Clustering $\propto 1/\alpha(k)$, where

$$\alpha(k,z) = \frac{2}{3}\frac{k^2 T(k) D(z)}{\Omega_m H_0^2}$$

satisfies

$$\delta_{\rm lin}(\mathbf{k},z) = \alpha(k,z)\Phi(\mathbf{k})$$



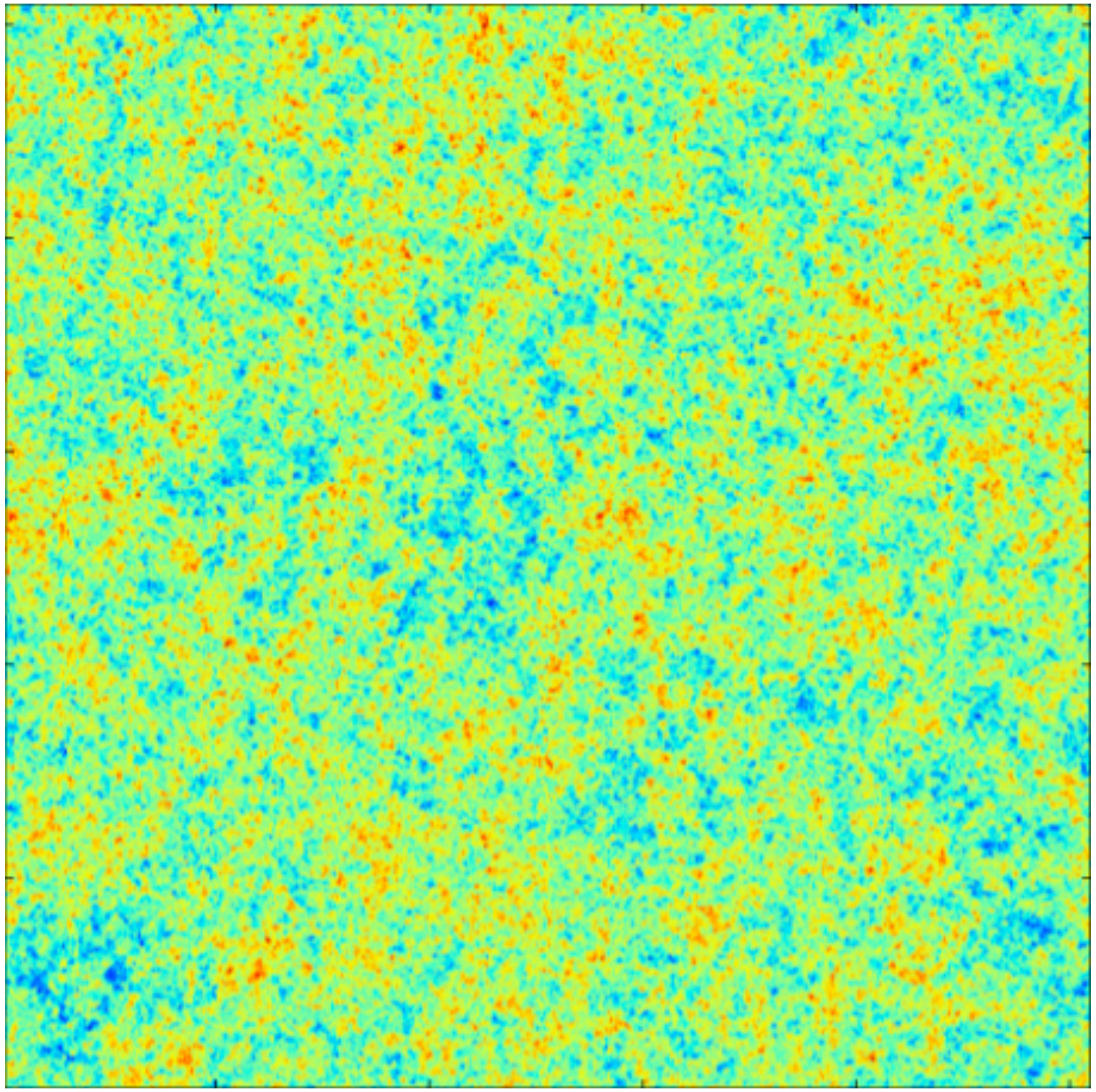
Dalal, Dore, Huterer & Shirokoff (2007)

f_{NL} cosmology: well-understood (both theoretically and in simulation)

g_{NL} cosmology:
Desjacques & Seljak (2010): analytic predictions for large-scale bias do not match simulations (?!)

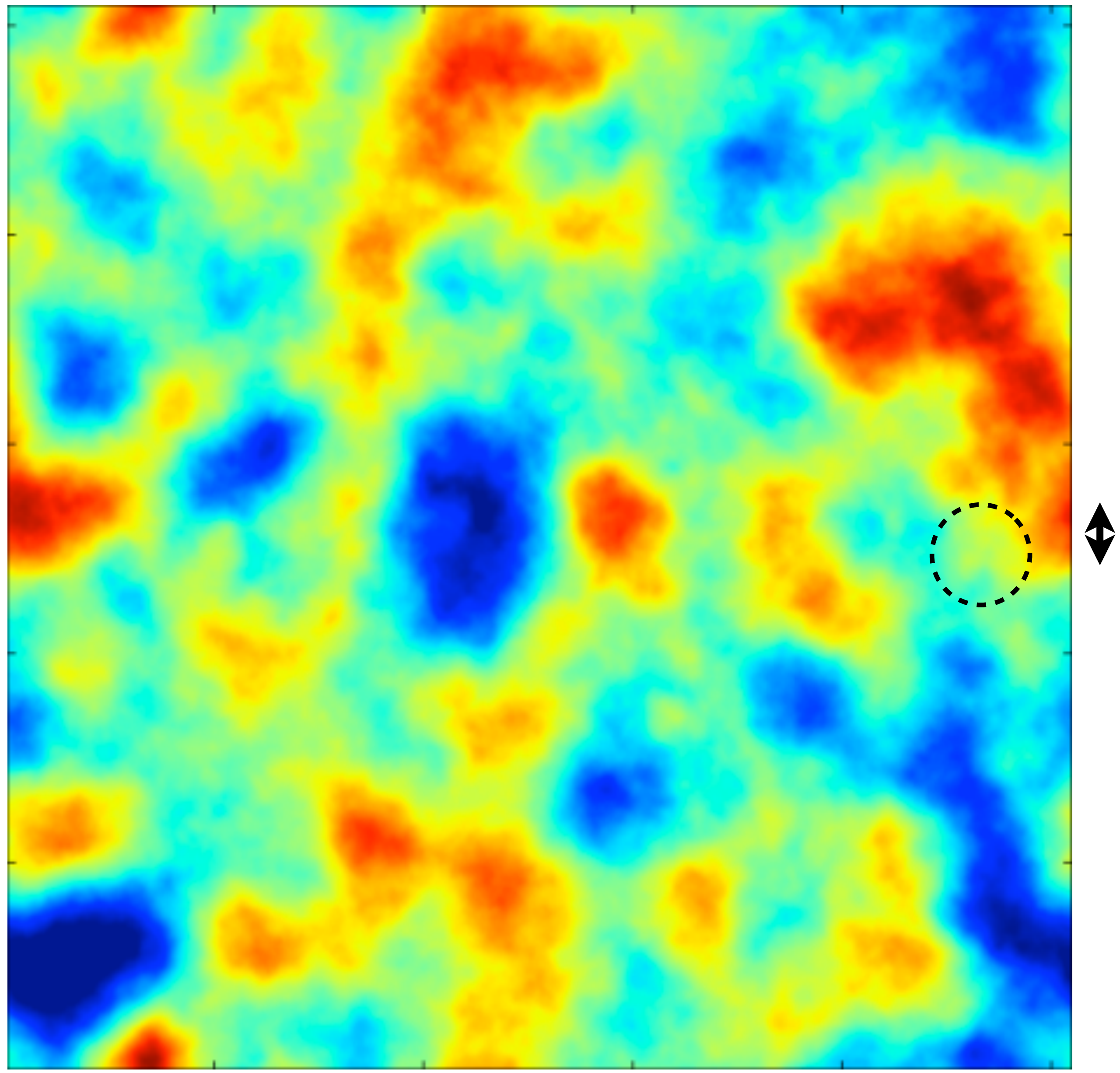
τ_{NL} cosmology:
Tseliakhovich & Hirata (2010): analytic prediction calculated, not compared to simulations

Press-Schechter Model



Start with *linear* density field $\delta_{\rm lin}(\mathbf{x}, z)$

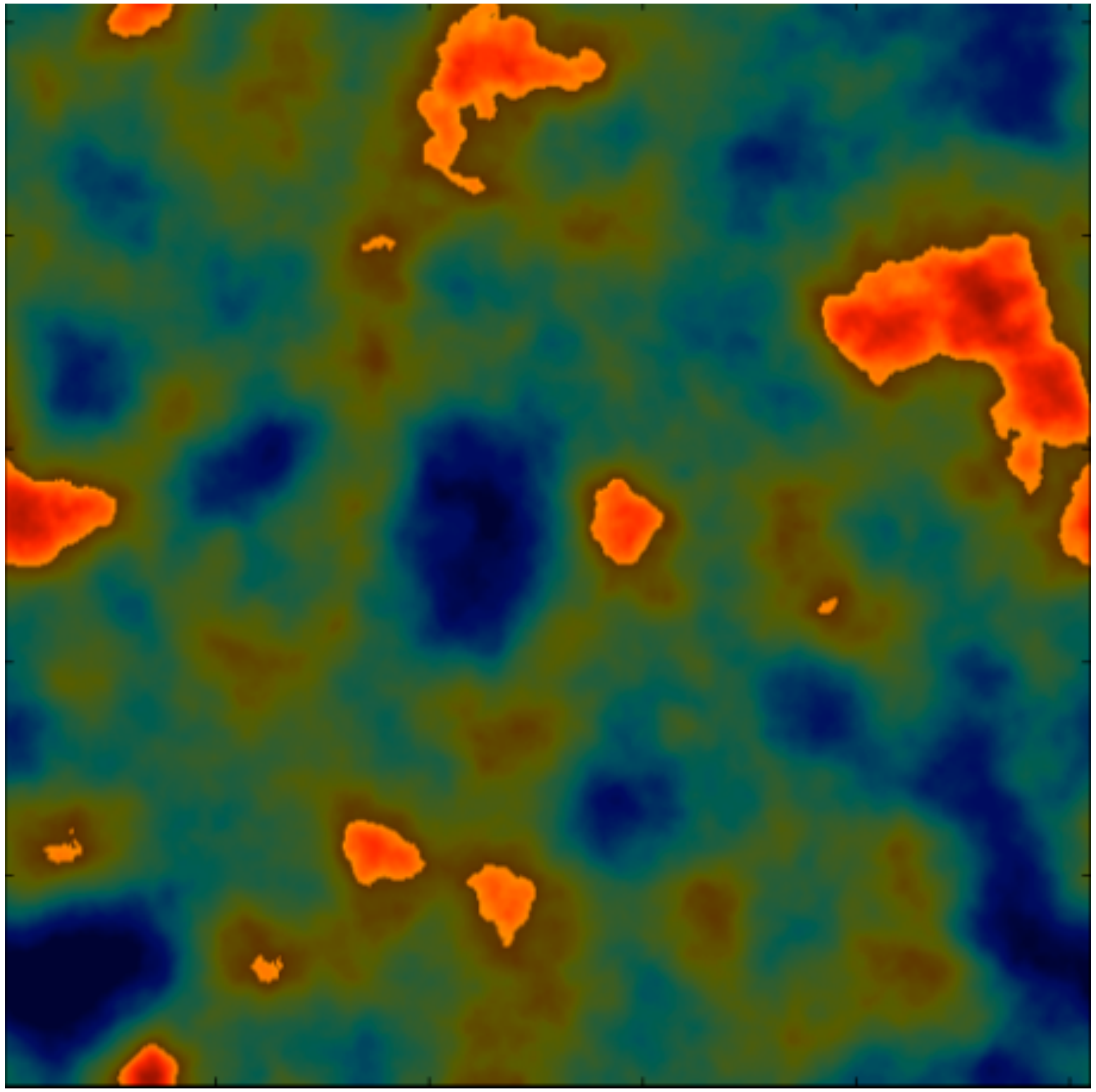
Press-Schechter Model



$$R = \left(\frac{3M}{4\pi \rho_m} \right)^{1/3}$$

Apply tophat smoothing on mass scale M to obtain smoothed linear density $\delta_M(\mathbf{x}, z)$

Press-Schechter Model



Apply threshhold: (halos of mass $\geq M$) $\Leftrightarrow$ (regions where $\delta_M(\mathbf{x}, z) \geq \delta_c$)

$\delta_c = 1.68$ motivated by analytic spherical collapse model

$\delta_c = 1.42$ gives better agreement with N-body simulations

Press-Schechter Model

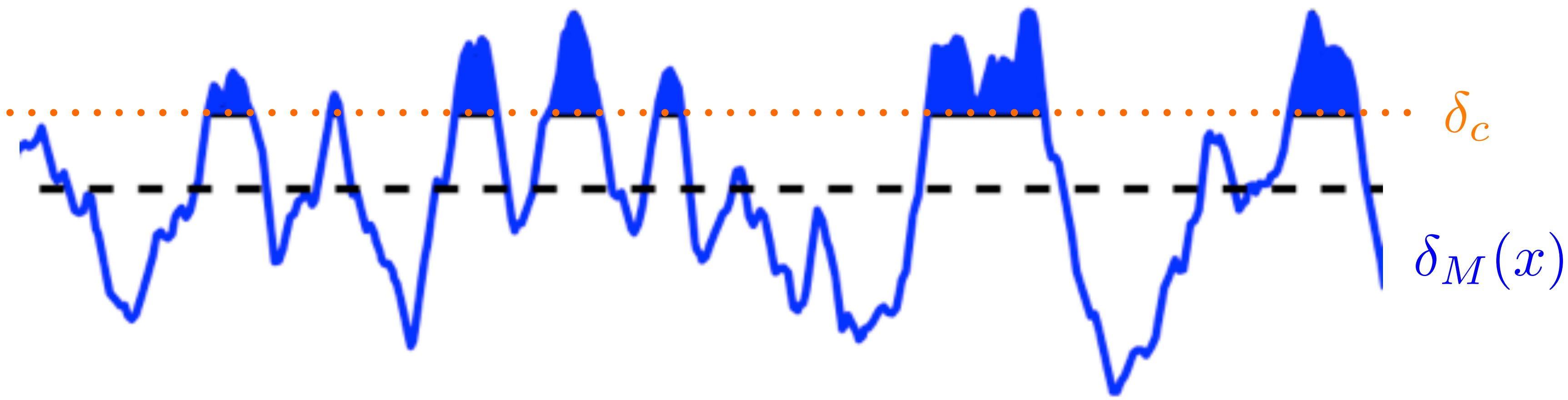
$$n_h = \begin{cases} \rho_m/M & \text{if } \delta_M(\mathbf{x}, z) \geq \delta_c \\ 0 & \text{if } \delta_M(\mathbf{x}, z) < \delta_c \end{cases}$$

[N.B.: This description omits some ingredients:

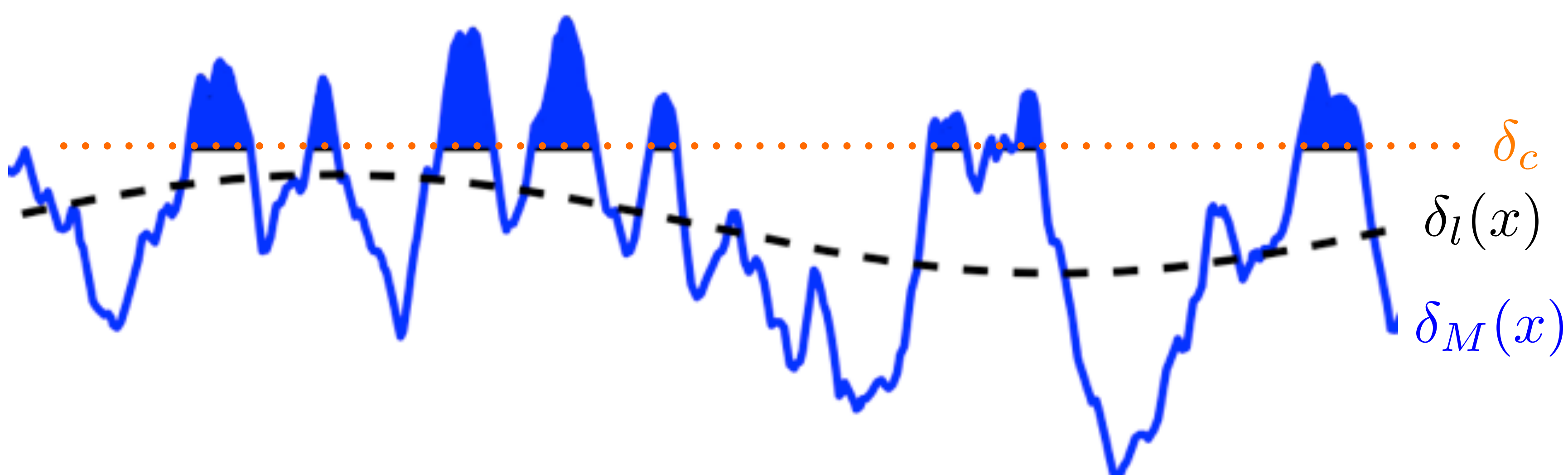
1) Lagrangian to Eulerian mapping
2) Poisson noise]

Large-scale halo bias: Gaussian case

Barrier crossing model: (halos of mass $\geq M$) $\Leftrightarrow$ (regions where $\delta_M \geq \delta_c$)



How is halo abundance affected by the presence of a long-wavelength overdensity $\delta_l(x)$?



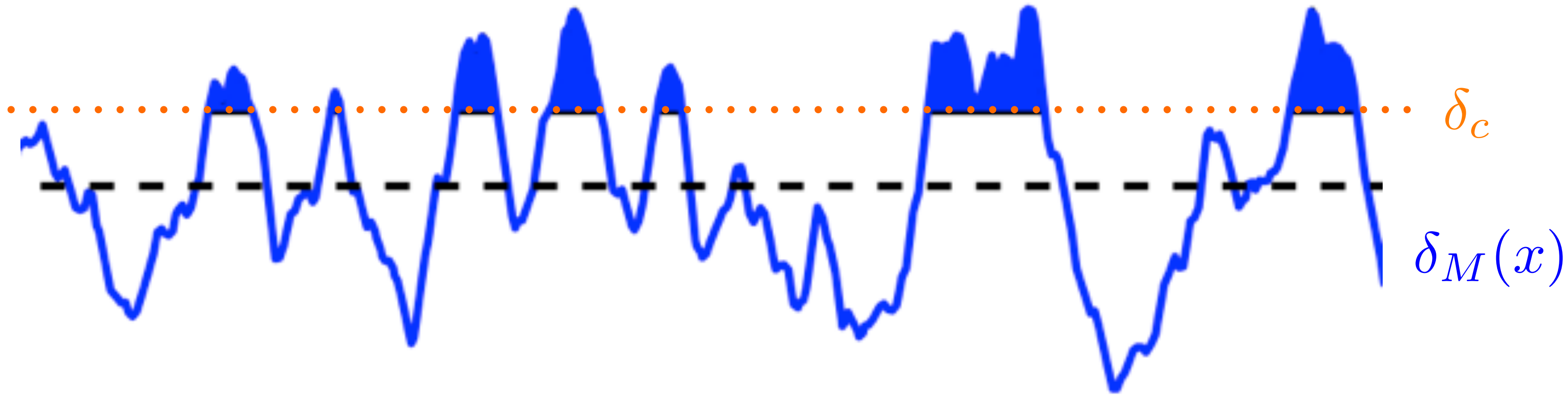
Local halo overdensity $\delta_h \approx b_0 \delta_l$ (where $b_0 = \dfrac{\partial \log n}{\partial \delta_l}$)

Define halo bias $b(k) = \dfrac{P_{mh}(k)}{P_{mm}(k)}$

$b(k) \to b_0$ $\quad$ (as $k \to 0$) $\quad$ ("weak" form of prediction)

$b_0 = \dfrac{\partial \log n}{\partial \delta_l}$ $\quad$ ("strong" prediction)

Large-scale bias: f_{NL} cosmology



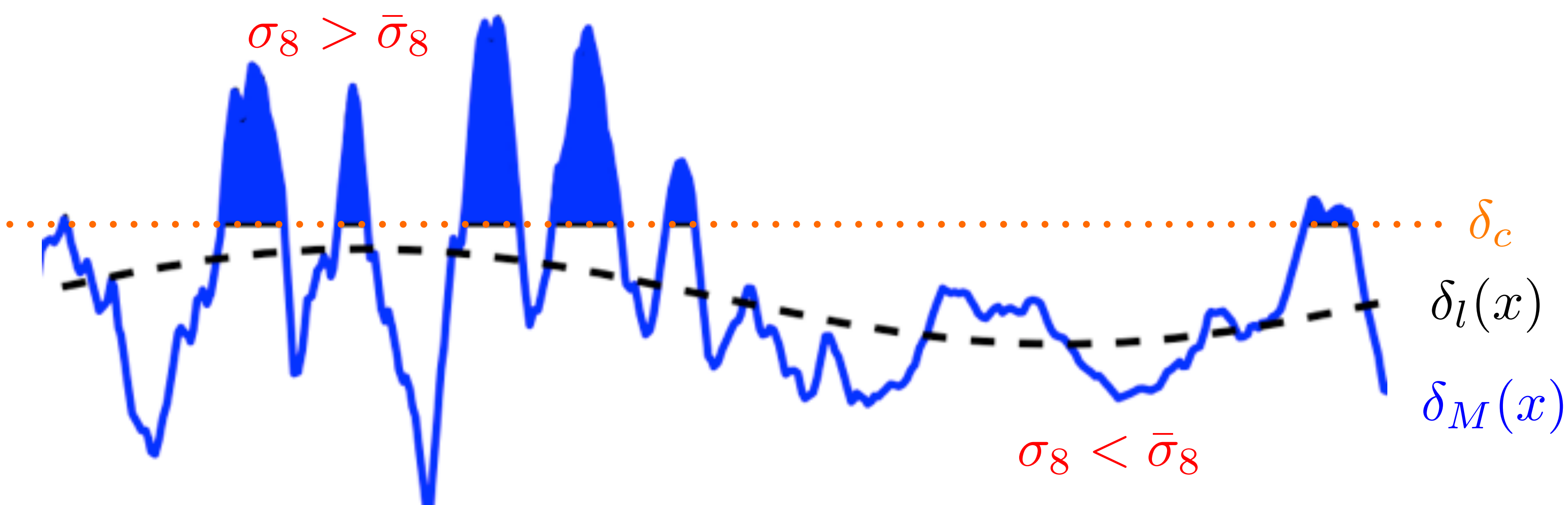
$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}(\Phi_G(\mathbf{x})^2 - \langle \Phi_G^2 \rangle)$$

Write $\Phi_G = \Phi_l + \Phi_s$

$$\Phi = \Phi_l + \underbrace{f_{NL}(\Phi_l^2 + \Phi_s^2 - \langle \Phi^2 \rangle)}_{\text{irrelevant for large-scale bias}} + \underbrace{(1 + 2 f_{NL} \Phi_l)\Phi_s}_{\text{Modulates ``local'' } \sigma_8}$$

irrelevant for large-scale bias

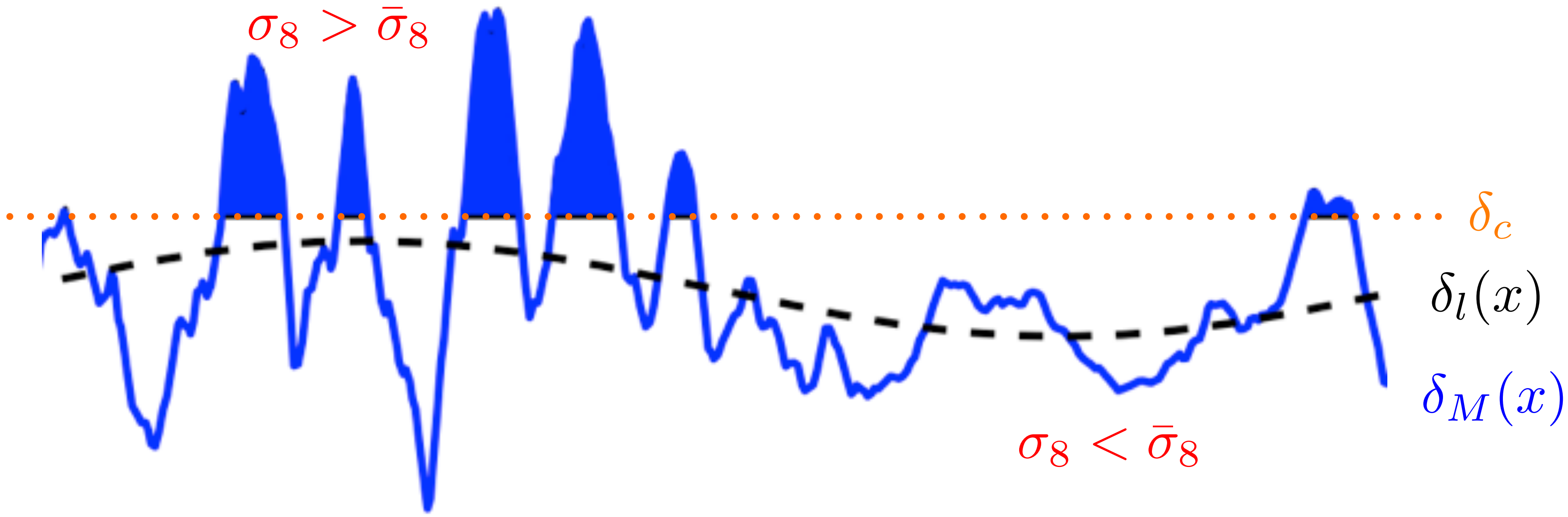
Modulates “local” σ_8:
$\sigma_8(x) = \bar{\sigma}_8(1 + 2 f_{NL} \Phi_l(x))$



Long-wavelength mode contributes to barrier crossing in two ways:

1) Contributes to the density fluctuation (as in a Gaussian cosmology)
2) Modulates “local σ_8” (new non-Gaussian effect, proportional to Φ_l rather than δ_l)

Large-scale bias: f_{NL} cosmology



Local halo overdensity contains two terms, corresponding to Gaussian + non-Gaussian contributions:

$$\delta_h \approx b_0 \delta_l + f_{NL} b_1 \Phi_l \qquad \left(b_0 = \frac{\partial \log n}{\partial \delta_l}, \;\; b_1 = 2 \frac{\partial \log n}{\partial \log \sigma_8} \right)$$

Halo bias $\;b(k) \rightarrow b_0 + f_{NL} \dfrac{b_1}{\alpha(k)} \quad (\text{as } k \rightarrow 0)$ ("weak" prediction)

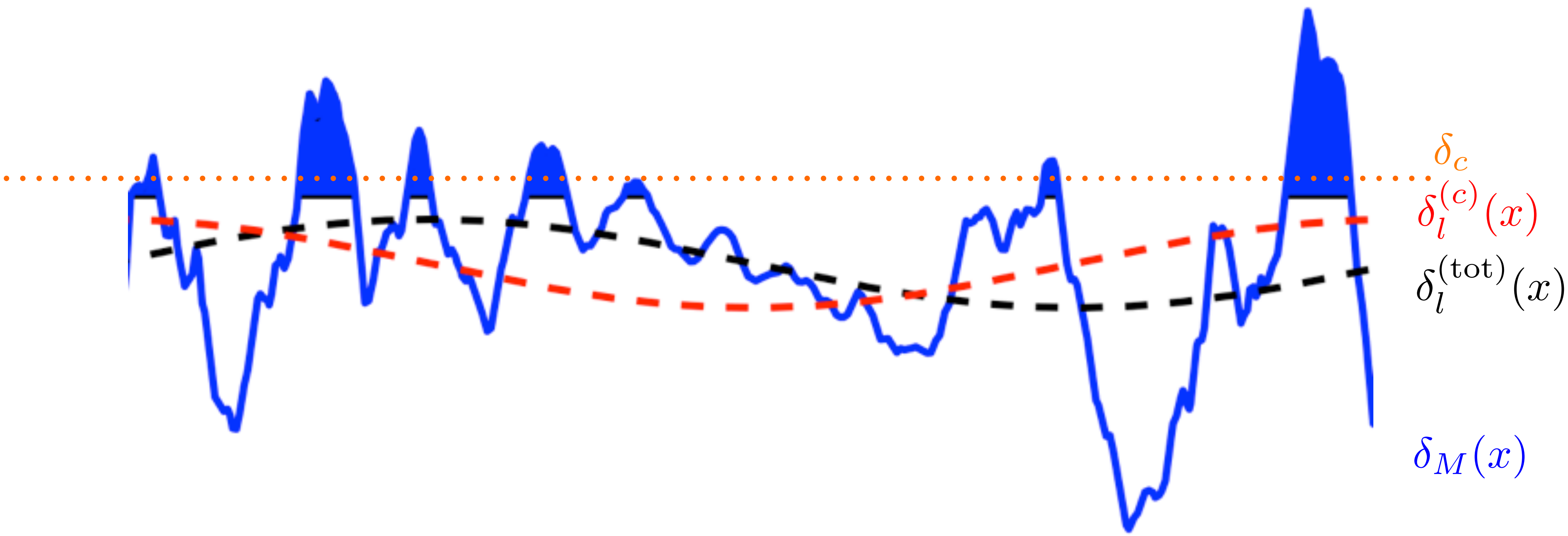
$b_1 = 2\delta_c(b_0 - 1)$ ("strong" prediction, assumes universal mass fn)

Large-scale bias: τ_{NL} cosmology

$$\Phi = (1-\alpha^2)^{1/2}\Phi_i + \alpha\Phi_c$$

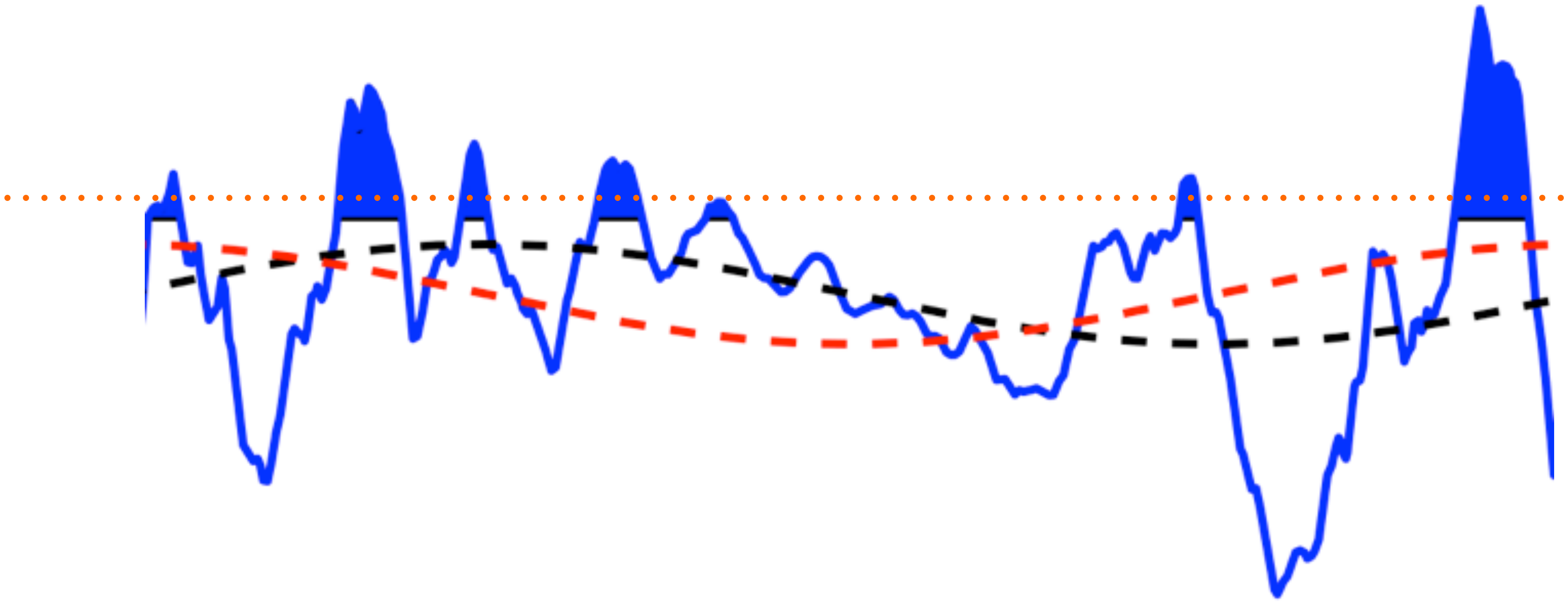
Contribution to barrier crossing due to long-wavelength mode:

1) Density fluctuation: proportional to total density $\delta_l^{(\mathrm{tot})}$
2) σ_8 modulation: proportional to curvaton part of potential $\Phi_l^{(\mathrm{c})}$



Gaussian clustering term follows the large-scale matter distribution $\delta_l^{(\mathrm{tot})}$
Non-Gaussian term is not 100% correlated

Large-scale bias: τ_{NL} cosmology



Local halo overdensity $\delta_h \approx b_0 \delta_l^{(\mathrm{tot})} + \frac{f_{NL}}{\alpha} b_1 \Phi_l^{(\mathrm{c})}$ $\left(b_0 = \frac{\partial \log n}{\partial \delta_l}, \quad b_1 = 2 \frac{\partial \log n}{\partial \log \sigma_8} \right)$

$$P_{mh}(k) = \left(b_0 + b_1 \frac{f_{NL}}{\alpha(k)} \right) P_{mm}(k)$$

$$P_{hh}(k) = \left(b_0^2 + 2 b_0 b_1 \frac{f_{NL}}{\alpha(k)} + b_1^2 \frac{(5/6)^2 \tau_{NL}}{\alpha(k)^2} \right) P_{mm}(k) + \frac{1}{n}$$

If $\tau_{NL} \neq \left(\frac{6}{5} f_{NL} \right)^2$, then halo bias is stochastic:

- (bias inferred from P_{hh}) $\neq$ (bias inferred from P_{mh})
- Halos and matter are not 100% correlated
- Halos of different masses are not 100% correlated with each other

Large-scale bias: g_{NL} cosmology

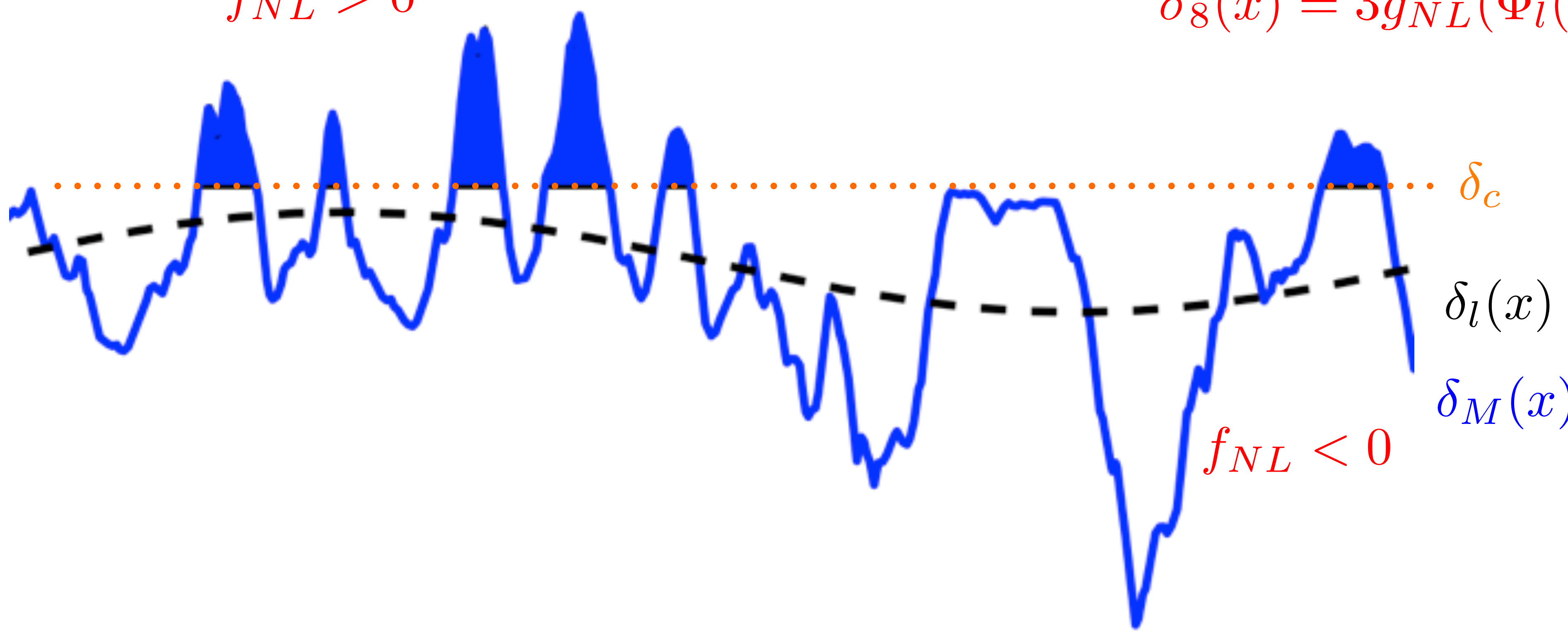
$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + g_{NL}(\Phi_G(\mathbf{x})^3 - 3\langle\Phi_G^2\rangle\Phi_G(\mathbf{x}))$$

$$\Phi = \Phi_l + \Phi_s + \underbrace{g_{NL}(\Phi_l^3 + \Phi_s^3 - 3\langle\Phi_l^2\rangle\Phi_l - 3\langle\Phi_s^2\rangle\Phi_s)}_{\text{irrelevant for large-scale bias}} + \underbrace{3g_{NL}\Phi_l(\Phi_s^2 - \langle\Phi_s^2\rangle)} + \underbrace{3g_{NL}(\Phi_l^2 - \langle\Phi_l^2\rangle)\Phi_s}$$

irrelevant for large-scale bias

Looks like spatially varying f_{NL}: $f_{NL}(x) = 3g_{NL}\Phi_l(x)$

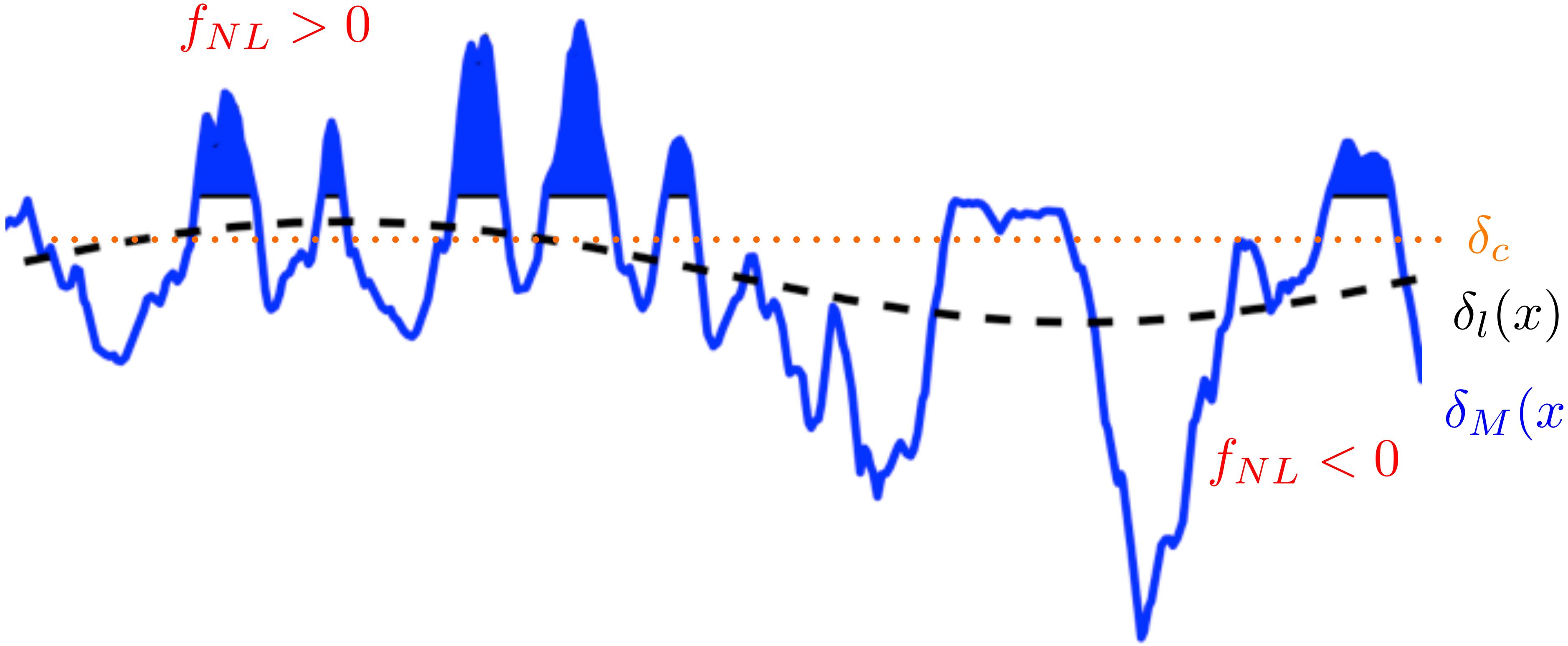
Looks like spatially varying σ_8: $\sigma_8(x) = 3g_{NL}(\Phi_l(x)^2 - \langle\Phi_l^2\rangle)\bar{\sigma}_8$



Long-wavelength mode contributes to barrier crossing in three ways:

1) Contributes to the density fluctuation (proportional to δ_l)
2) Modulates "local f_{NL}" (proportional to Φ_l)
3) Modulates "local σ_8" (proportional to Φ_l^2)

Large-scale bias: g_{NL} cosmology



Neglecting third contribution, local halo overdensity consists of two terms:

$$\delta_h \approx b_0 \delta_l + g_{NL} b_2 \Phi_l$$

Halo bias $b(k) \to b_0 + g_{NL} \dfrac{b_2}{\alpha(k)} \quad (\text{as } k \to 0)$ (weak prediction)

$$b_2 = 3 \left(\frac{\partial \log n}{\partial f_{NL}} \right)$$ (stronger)

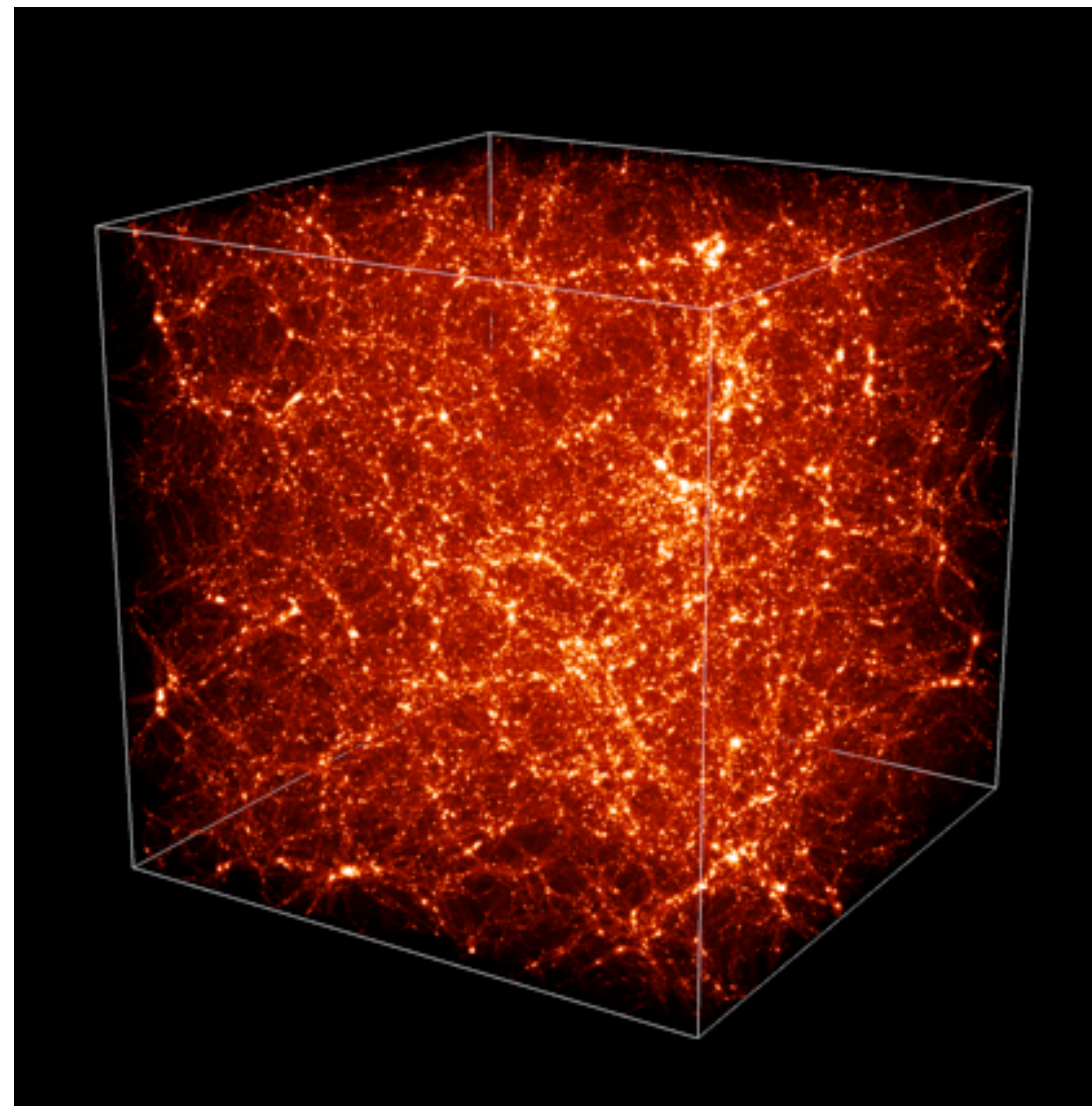
$$= \frac{\kappa_3(M)}{2} H_3\left(\frac{\delta_c}{\sigma(M)}\right) - \frac{d\kappa_3/dM}{d\sigma/dM} \frac{\sigma(M)^2}{2\delta_c} H_2\left(\frac{\delta_c}{\sigma(M)}\right)$$ (strongest)

N-body simulations

Collisionless N-body simulations, GADGET-2 TreePM code.

Unless otherwise specified:

- periodic boundary conditions, $L_{\rm box} = 1600\ h^{-1}\ {\rm Mpc}$
- particle count $N = 1024^3$
- force softening length $R_s = 0.05\,(L_{\rm box}/N^{1/3})$
- initial conditions simulated at $z_{\rm ini} = 100$ using Zeldovich approximation
- FOF halo finder, link length $L_{\rm FOF} = 0.2\,(L_{\rm box}/N^{1/3})$

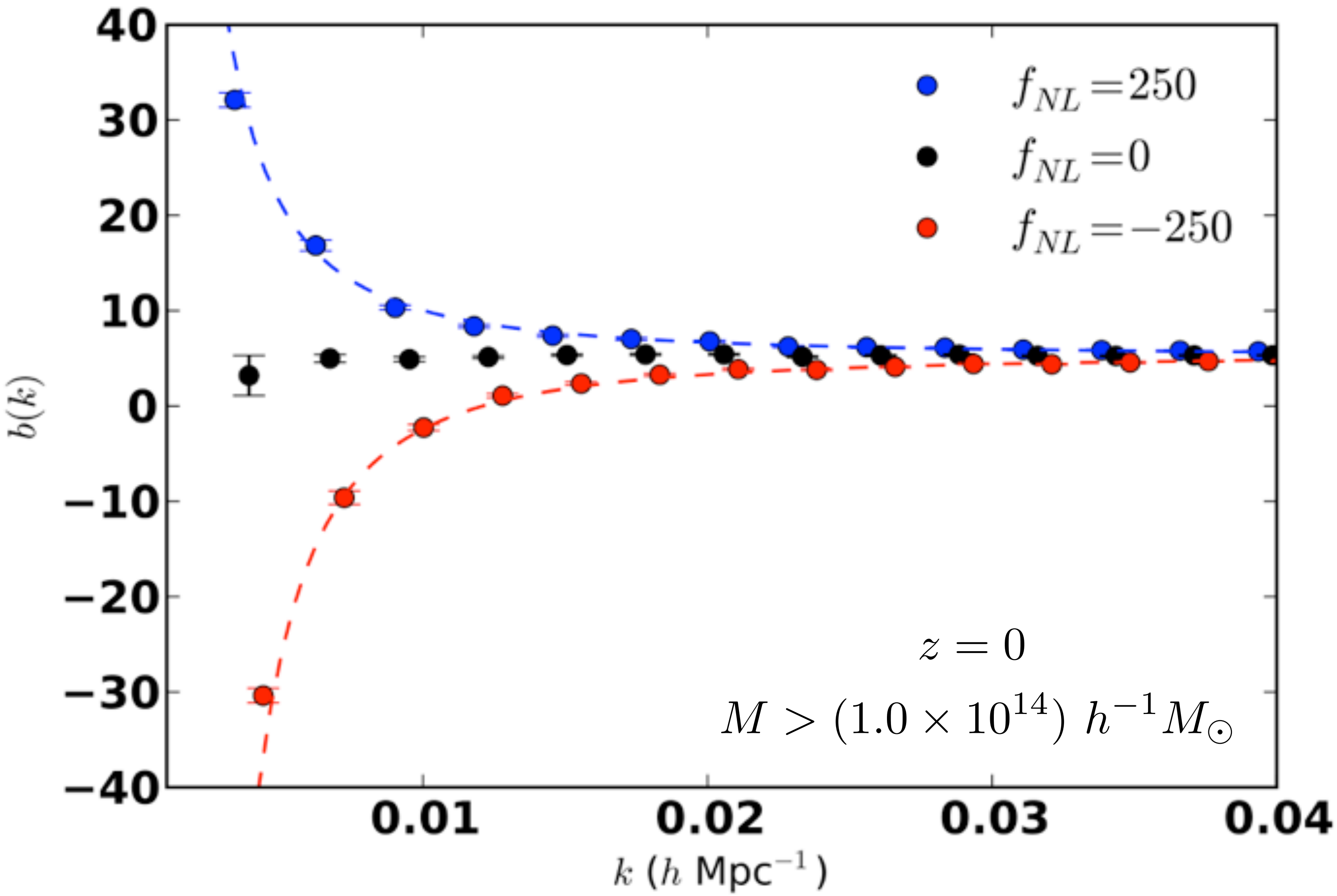


Halo bias: f_{NL} simulations

Prediction from barrier crossing model:

$$b(k) \to b_0 + f_{NL} \frac{b_1}{\alpha(k)} \qquad\qquad b_1 = 2\delta_c(b_0 - 1)$$

Agreement with simulations: perfect!



Stochastic halo bias: τ_{NL} simulations

Define stochasticity $r(k)$ by:

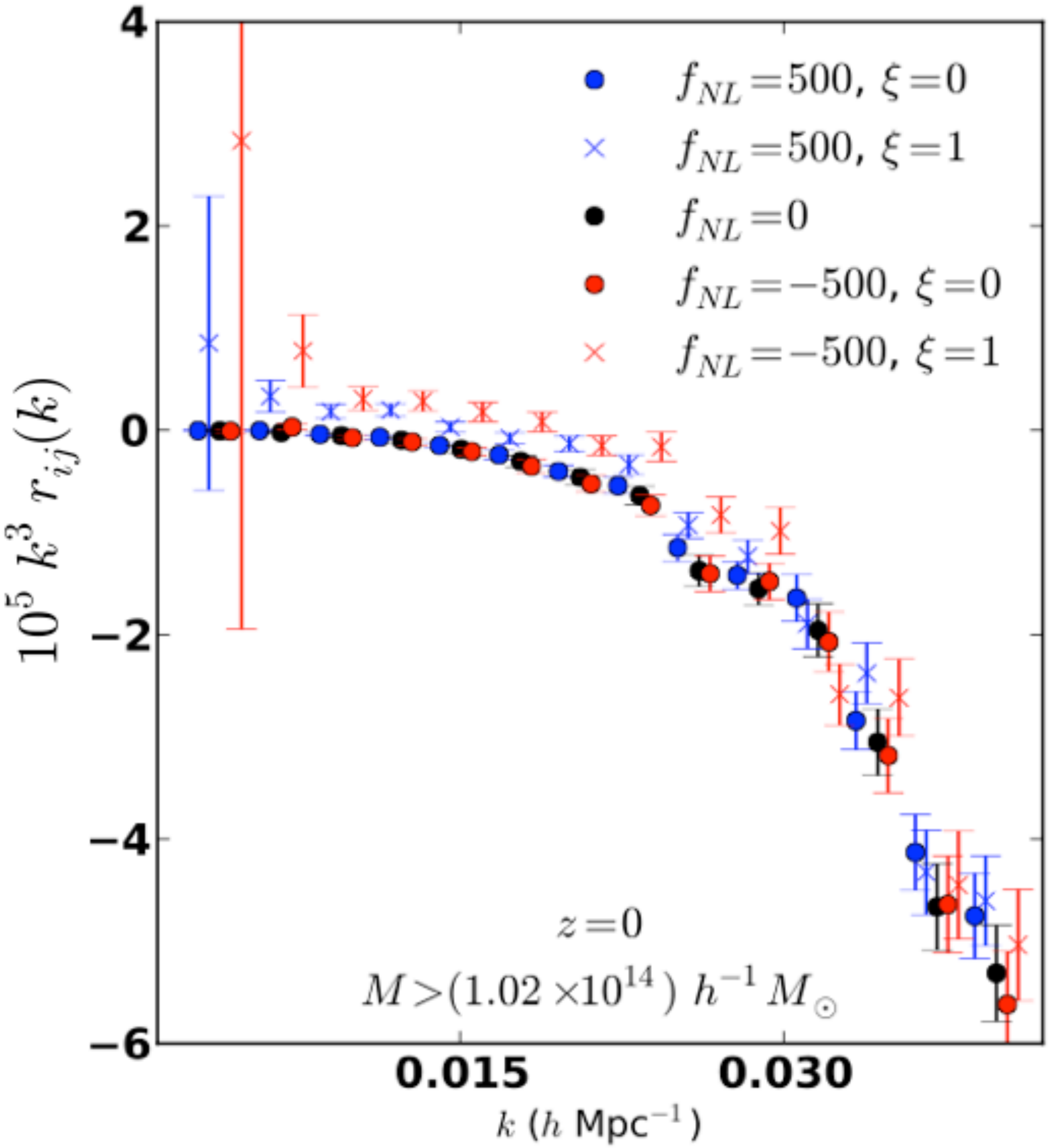
$$r(k) = \frac{P_{hh}(k) - 1/n}{P_{mm}(k)} - \left(\frac{P_{mh}(k)}{P_{mm}(k)}\right)^2$$

Prediction from barrier crossing model:

$$r(k) = \left[\left(\frac{5}{6}\right)^2 \tau_{NL} - f_{NL}^2\right] \frac{b_1^2}{\alpha(k)^2}$$

Results from simulations:

- significant stochasticity in Gaussian cosmology
- no change to stochasticity in f_{NL} cosmology
- boosted stochasticity in τ_{NL} cosmology



Smith & Loverde (2010)

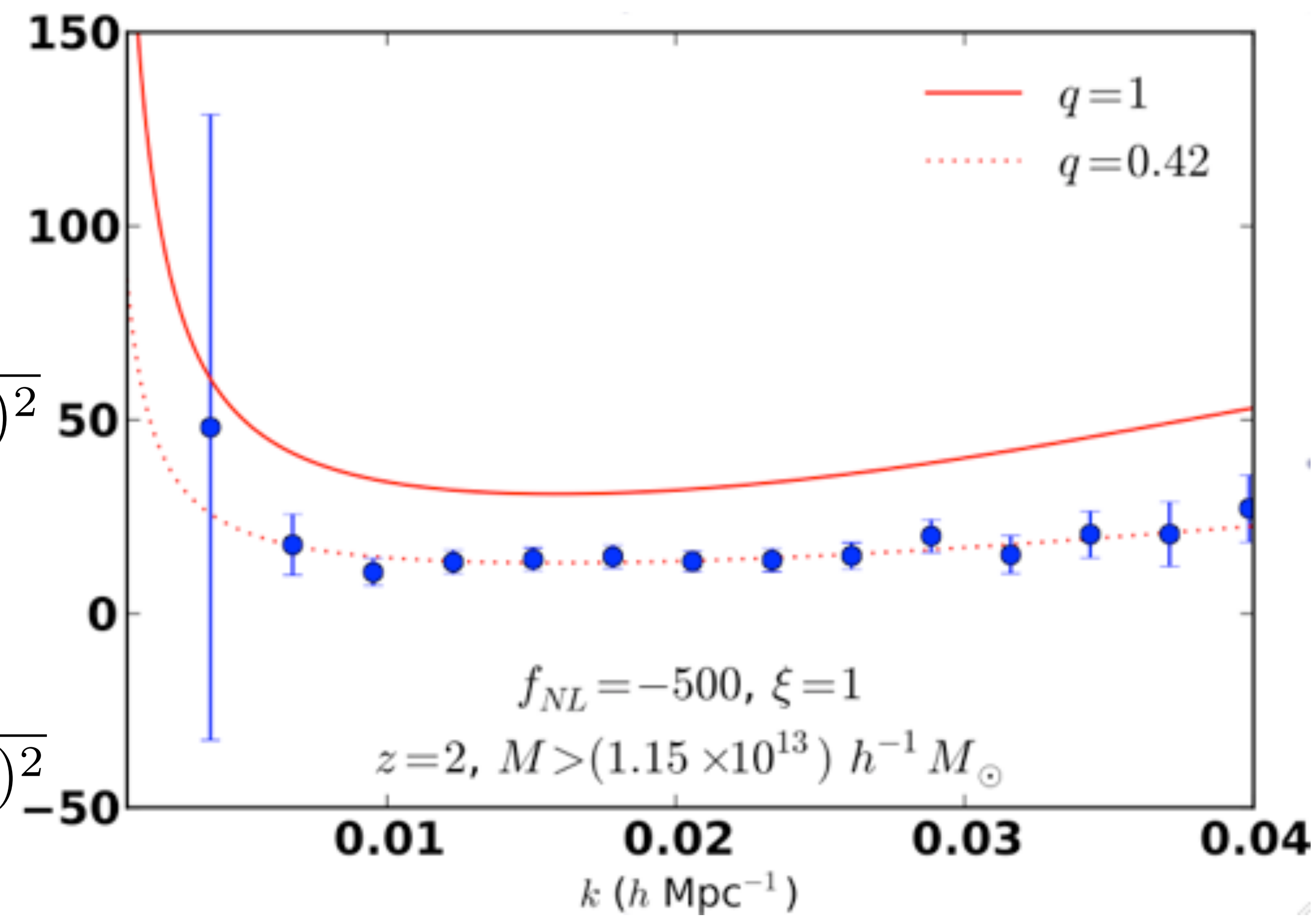
Stochastic halo bias: τ_{NL} simulations

Interpret barrier crossing result as prediction for $r_{NG}(k) - r_G(k)$, i.e. non-Gaussian contribution

$$r_{NG}(k) - r_G(k) = \left[\left(\frac{5}{6}\right)^2 \tau_{NL} - f_{NL}^2 \right] \frac{b_1^2}{\alpha(k)^2}$$

Comparison with simulations: shape is correct, amplitude is not!

$$r_{NG}(k) - r_G(k) = q \left[\left(\frac{5}{6}\right)^2 \tau_{NL} - f_{NL}^2 \right] \frac{b_1^2}{\alpha(k)^2}$$



	Mass range ($h^{-1}M_\odot$)	$f_{NL}=500$	$f_{NL}=250$	$f_{NL}=-250$	$f_{NL}=-500$
$z=2$	$M>1.15\times10^{13}$	0.98 ± 0.07	0.88 ± 0.08	0.62 ± 0.06	0.42 ± 0.03
$z=1$	$1.15\times10^{13}<M<2.32\times10^{13}$	0.79 ± 0.09	0.83 ± 0.12	0.67 ± 0.09	0.46 ± 0.04
	$M>2.32\times10^{13}$	0.83 ± 0.07	0.70 ± 0.08	0.66 ± 0.07	0.51 ± 0.04
$z=0.5$	$1.15\times10^{13}<M<2.32\times10^{13}$	1.01 ± 0.18	0.92 ± 0.29	0.45 ± 0.19	0.57 ± 0.10
	$2.32\times10^{13}<M<4.66\times10^{13}$	0.80 ± 0.15	0.58 ± 0.22	0.73 ± 0.19	0.48 ± 0.08
	$M>4.66\times10^{13}$	0.81 ± 0.09	0.79 ± 0.12	0.80 ± 0.10	0.51 ± 0.05
$z=0$	$1.15\times10^{13}<M<2.32\times10^{13}$	1.37 ± 0.80	1.06 ± 1.12	1.00 ± 1.41	0.90 ± 0.51
	$2.32\times10^{13}<M<4.66\times10^{13}$	1.35 ± 0.44	1.57 ± 0.77	0.82 ± 0.59	0.58 ± 0.25
	$4.66\times10^{13}<M<1.02\times10^{14}$	0.71 ± 0.26	0.90 ± 0.49	1.12 ± 0.41	0.63 ± 0.17
	$M>1.02\times10^{14}$	0.79 ± 0.13	0.93 ± 0.21	0.73 ± 0.15	0.53 ± 0.07

Table 3: Values of the q-parameter, defined in Eq. (35), obtained from N-body simulations for various values of f_{NL}, redshift, and mass bin. (We take $\xi=1$ throughout)

Halo bias: g_{NL} simulations

Predictions from barrier crossing model:

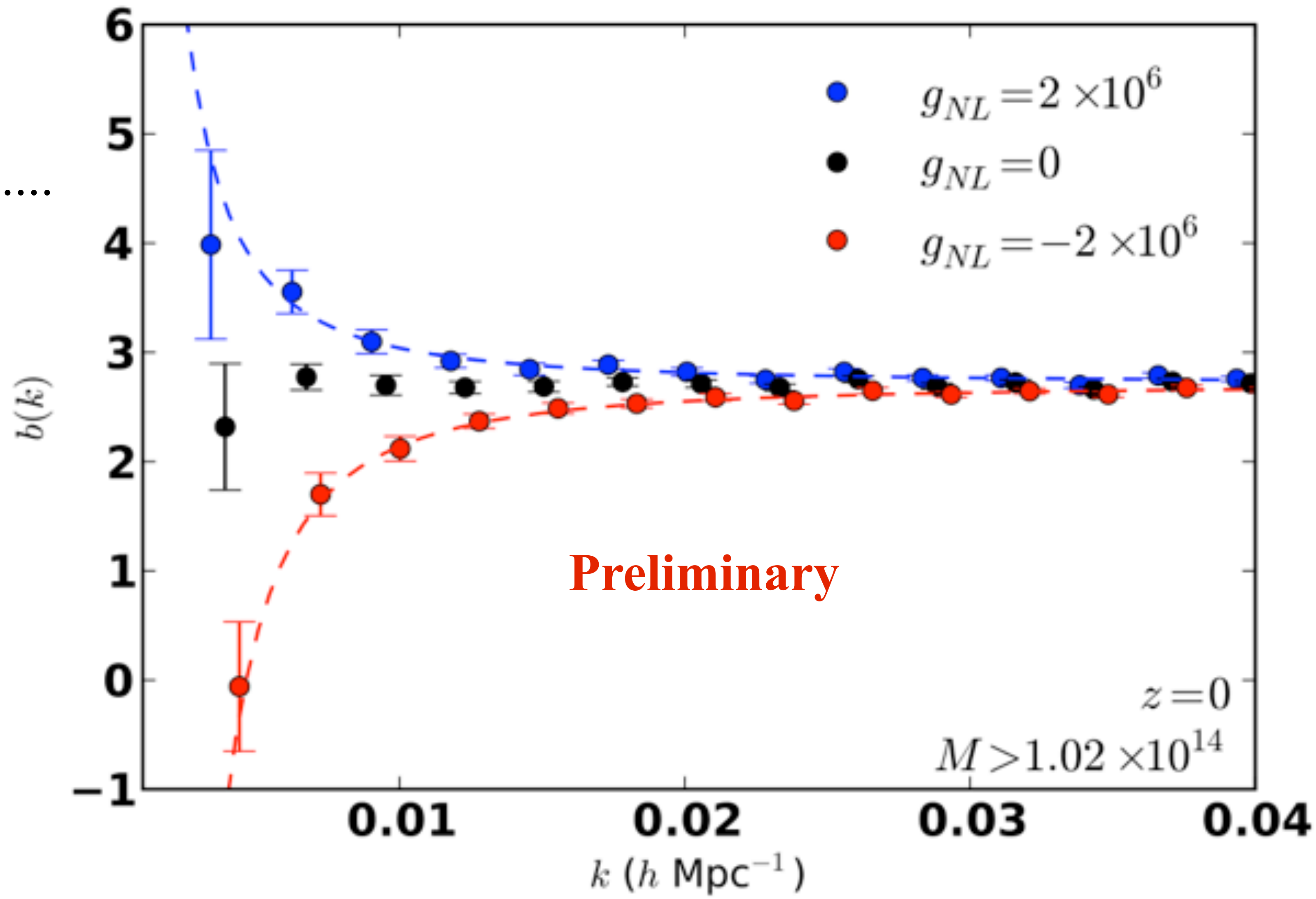
$$b(k) \to b_0 + g_{NL} \frac{b_2}{\alpha(k)}$$

$$\begin{aligned} b_2 &= 3\left(\frac{\partial \log n}{\partial f_{NL}}\right) \\ &= \frac{\kappa_3(M)}{2} H_3\left(\frac{\delta_c}{\sigma(M)}\right) - \frac{d\kappa_3/dM}{d\sigma/dM} \frac{\sigma(M)^2}{2\delta_c} H_2\left(\frac{\delta_c}{\sigma(M)}\right) \end{aligned}$$

Let's test this prediction in several steps.....

First: is $b(k) = b_0 + g_{NL} \frac{b_2}{\alpha(k)}$ a good fit, treating b_0, b_2 as free parameters?

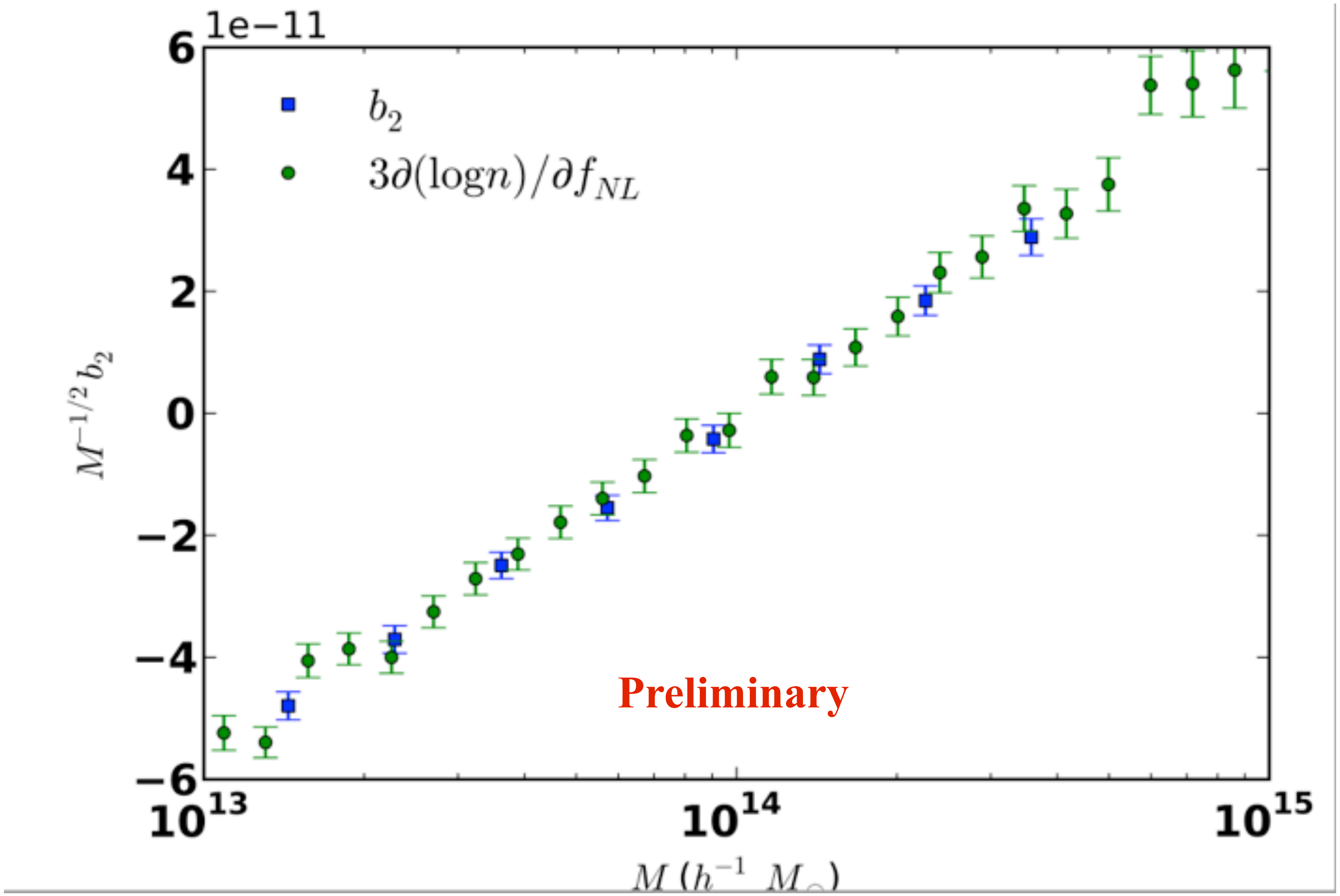
Answer: yes!



Halo bias: g_{NL} simulations

Second: general relation between g_{NL} dependence of bias and f_{NL} dependence of mass function

$$b_2 = 3\left(\frac{\partial \log n}{\partial f_{NL}}\right)$$

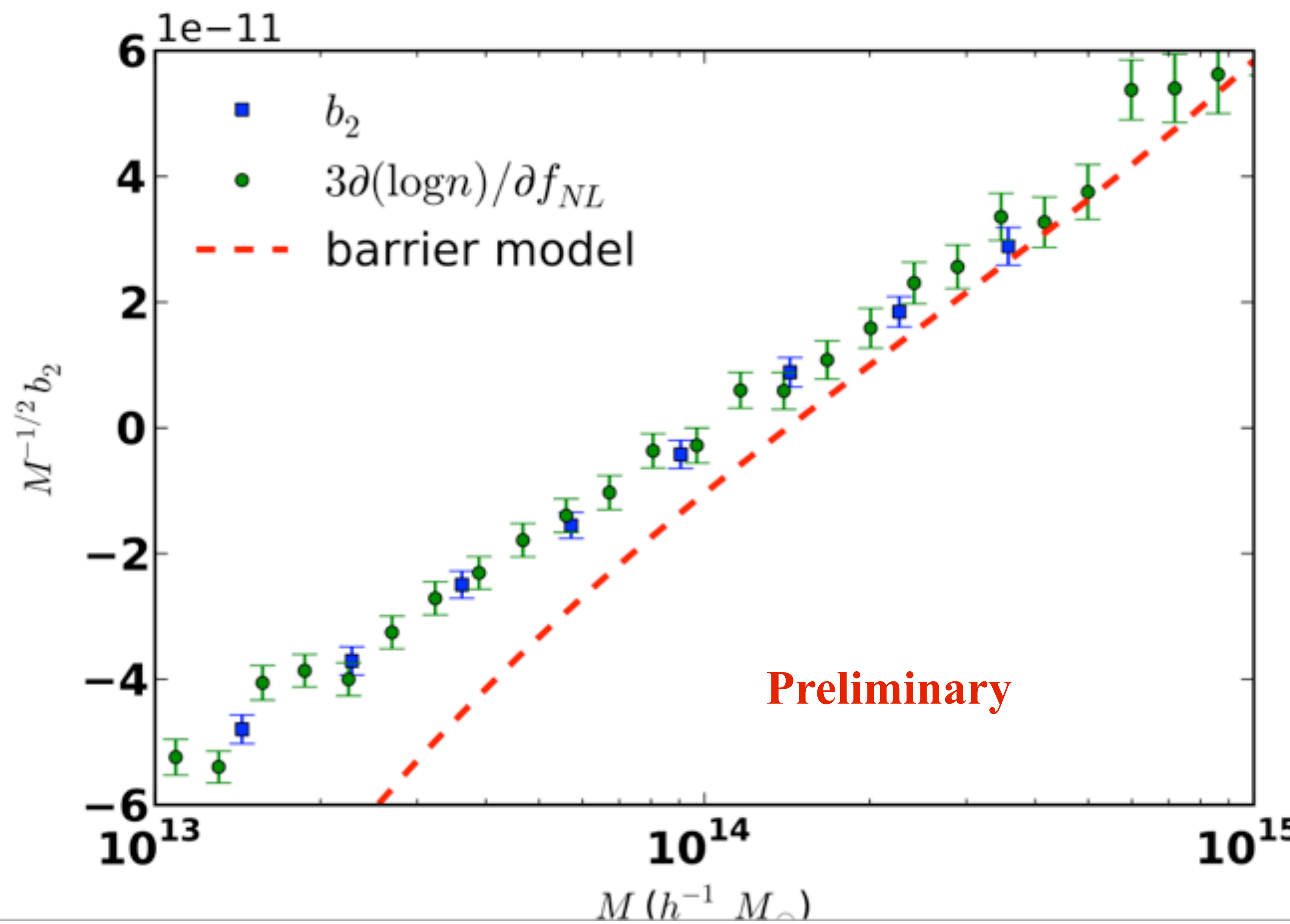


Agreement with simulations: perfect!

Halo bias: g_{NL} simulations

Third: barrier crossing model prediction for b_2 :

$$\frac{\kappa_3(M)}{6} H_3\left(\frac{\delta_c}{\sigma(M)}\right) - \frac{d\kappa_3/dM}{d\sigma/dM} \frac{\sigma(M)^2}{6\delta_c} H_2\left(\frac{\delta_c}{\sigma(M)}\right)$$



Works well for large halo mass (most relevant for observations); breaks down at low mass

Conclusions

- Analytic models (peak-background split, barrier crossing) can qualitatively describe halo clustering for a generalized local non-Gaussianity with parameters $\{f_{NL}, g_{NL}, \tau_{NL}\}$

- Minor puzzle: barrier crossing prediction for g_{NL} bias breaks down at low halo mass

- More significant puzzle: understanding amplitude of stochasticity in τ_{NL} cosmology